EVALUATING THE LOWER BOUND FOR THE CRITICAL FORCES IN THE SHOCK LOADING OF A ROD

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Rods may experience substantially greater loads under shock loading (without losing stability) than under static loading, depending on the loading time. Evaluating stability during shock loading involves selection of a stability criterion which is a function of the operating regime of the structure and manufacturing tolerances.

Aspects of the stability of rods under shock loading were examined in [1-7].

It should be noted that numerical solutions of problems on loss of stability under shock loading [2, 3] runs into a difficulty with can be circumvented theoretically but which is always present in the design of actual structures. This problem has to do with fixing the initial deviations of the rod, although the actual initial deviation is a random variable which in practice can only be assigned within a certain range — the manufacturing tolerance.

This problem has also not been solved in studies using analytical methods. The investigations [1, 6, 7] do not report any dependence of the critical forces on manufacturing tolerances.

The present study evaluates the lower bounds for the critical force in shock loading for a certain special formulation. Here, although the initial deflection of the rod is a random variable and the critical force should be of a statistical character, it turns out that it is possible to obtain a deterministic evaluation for the minimum critical force.

1. We will examine the problem of the stability of a hinged rod of length l compressed by an impulsive load which is constant up to a certain time t_0 and is then removed.

The rod has the initial deflection U(x). It is assumed that the load N_0 exceeds the critical Eulerian load and that the time t_0 is significantly greater than the time of propagation of longitudinal waves in the rod.

With such a formulation, the problem reduces to examining a well-known equation for transverse vibrations of a rod

$$EJW_{,xxxx} + \rho SW_{,tt} + NW_{,xx} + NU_{,xx} = 0, \qquad (1.1)$$
$$0 < x < l, \ 0 < t < \infty;$$
$$W = 0, \ W_{,t} = 0, \ t = 0; \qquad (1.2)$$
$$W = 0, \ W_{,xx} = 0, \ x = 0, \ x = l; \qquad (1.3)$$

$$N = \begin{cases} N_0, & 0 < t < t_0, \\ 0, & t_0 < t < \infty, \end{cases} \qquad (1.4)$$

where U(x) is a random function describing the initial deviation of the rod from rectilinearity (due to the manufacturing technology), on which we impose the limitation

$$|U(x)| \leqslant \varepsilon_0, \quad \varepsilon_0 > 0. \qquad (1.5)$$

The value of ε_0, determining the manufacturing tolerance for the product, is assigned. In Eq. (1.1), E, J, ρ, and S are the elastic modulus, moment of inertia, density of the material, and cross sectional area of the rod, respectively.

In changing over to dimensionless variables and parameters, with allowance for conditions (1.2)-(1.4) we represent Eq. (1.1) in the form

$$W_{,\xi\xi\xi\xi} + a^2W_{,\tau\tau} + b^2W_{,\xi\xi} + b^2U_{,\xi\xi} = 0, \quad 0 < \xi < 1, \ 0 < \tau < \infty; \qquad (1.6)$$

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$$W = W_{,\tau} = 0,\ \tau = 0;\ W = W_{,\xi\xi} = 0,\ \xi = 0,\ \xi = 1;$$
$$\xi = x/l,\ \tau = tE^{1/2}l^{-1}\rho^{-1/2},\ a^2 = sl^2J^{-1}; \tag{1.7}$$

$$b^2 = \begin{cases} \eta\pi^2, & 0 < \tau < \tau_0, \\ 0, & \tau_0 < \tau < \infty, \end{cases} \tag{1.8}$$
$$\eta = N_0/N_e,\quad \tau_0 = t_0E^{1/2}l^{-1}\rho^{-1/2},$$

where $N_e = \pi^2 l^{-2} EJ$ is the critical Eulerian load. As one possible criterion of stability we take a criterion connected with the limitation on the lateral displacement of the rod:

$$\max |W(U, \tau, \eta)| \leqslant W_0,\ |U(x)| < \varepsilon_0,\ 0 < \tau < \infty,$$
$$W_0 \gg \varepsilon_0 > 0.$$

Here, a value of η for which the absolute inequality is satisfied is taken as a subcritical value, while a value of η^* for which the equality is satisfied is a critical value.

2. With expansion of the function U(x) into a Fourier series in functions $\sin k\pi\xi$ on the interval $0 < \xi < 1$

$$U = \sum_1^\infty C_k \sin k\pi\xi,\ C_k = 2\int_0^1 U(\xi) \sin k\pi\xi d\xi. \tag{2.1}$$

We obtain the following estimate from (1.5) and (2.1) for the coefficients C_k, which are random variables

$$|C_k| \leqslant 2\int_0^1 |U(\xi)|\,|\sin k\pi\xi|\,d\xi \leqslant 2\varepsilon_0. \tag{2.2}$$

We seek a solution of Eq. (1.6) in the form of a series

$$W = \sum_1^\infty C_k T_k(\tau) \sin k\pi\xi; \tag{2.3}$$

$$T_k(0) = T_{k,\tau}(0) = 0. \tag{2.4}$$

Initial and boundary conditions (1.7) are satisfied when (2.3) and (2.4) are chosen for the form of the solution.

With insertion of (2.3) into Eq. (1.6), for each function $T_k(\tau)$ there is the equation

$$T_{k,\tau\tau} + (k\pi)^2a^{-2}[(k\pi)^2 - b^2]T_k - (k\pi)^2b^2a^{-2} = 0. \tag{2.5}$$

It follows from analysis of Eq. (2.5) that with a fixed value of N_0 there is a finite number of values of $k \leqslant m_0$ when the following condition is satisfied

$$\varkappa \equiv (k\pi)^2 - b^2 < 0, \tag{2.6}$$

which is satisfied by the quantity

$$m_0 = [\sqrt{\eta}]. \tag{2.7}$$

Condition (2.6) means that, besides vibrations corresponding to $\varkappa > 0$, there is motion with an exponential increase in amplitude at $1 \leqslant k \leqslant m_0$. This motion is mainly responsible for the curvature of the rod, i.e., in the displacement

$$W = \sum_{k=1}^\infty C_kT_k(\tau) \sin k\pi\xi \equiv \sum_{k=1}^{m_0} C_kT_k(\tau) \sin k\pi\xi + \sum_{m_0+1}^\infty C_kT_k(\tau) \sin k\pi\xi$$

we can ignore the second sum, which is small compared to the first, having terms which increase without limit:

$$W \approx \sum_{k=1}^{m_0} C_kT_k(\tau) \sin k\pi\xi. \tag{2.8}$$

Such a division of the displacements is characteristic of studies of stability under shock loading [1, 6, 7]. Some of the investigations, moreover, isolate and retain only certain basic terms of the sum (2.8).

We will seek the solution of Eq. (2.5), with initial conditions (2.4), in the class of continuous functions having continuous first derivatives. This means that there will be no sudden changes in the displacement and rate of displacement. With allowance for this and

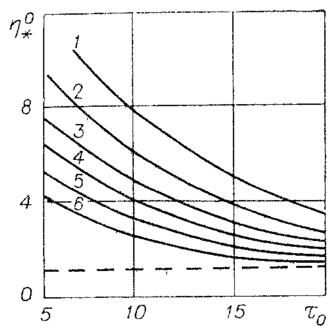

Fig. 1

the discontinuity of the function $b(\tau)$, the solution of (2.5) for $k \leqslant m_0$ has the form

$$T_k(\tau) = -b_k a_k^{-2}[1 - \operatorname{ch} a_k\tau], \quad 0 \leqslant \tau \leqslant \tau_0,$$
$$T_k(\tau) = -A_1 \cos c_k\tau - A_2 \sin c_k\tau, \ \tau_0 < \tau < \infty,$$
$$a_k = k\pi a^{-1}[b^2 - (k\pi)^2]^{1/2}, \ b_k = (k\pi)^2 b^2 a^{-2}, \quad c_k = k^2\pi^2 a^{-1},$$
$$A_1 = b_k a_k^{-2}[1 - \operatorname{ch} a_k\tau_0] \cos c_k\tau_0 + b_k a_k^{-1} c_k^{-1} \operatorname{sh} a_k\tau_0 \sin c_k\tau_0,$$
$$A_2 = b_k a_k^{-2}[1 - \operatorname{ch} a_k\tau_0] \sin c_k\tau_0 - b_k a_k^{-1} c_k^{-1} \operatorname{sh} a_k\tau_0 \cos c_k\tau_0.$$

Here, for the functions $T_k(\tau)$ we obtain the estimate

$$\max_{0<\tau<\infty} |T_k(\tau)| = \max_{\tau_0<\tau<\infty} |T_k(\tau)| \leqslant |A_1| + |A_2| \leqslant F_k(\tau_0, \eta, a) \equiv \equiv 2b_k a_k^{-2}[\operatorname{ch} a_k\tau_0 - 1] + 2b_k a_k^{-1} c_k^{-1} \operatorname{sh} a_k\tau_0. \quad (2.9)$$

The function $F_k(\tau_0, \eta, a)$, as a function of the parameter η, is monotonically increasing because there is a monotonic increase in the factor

$$b_k(\eta), \quad a_k^{-2}[\operatorname{ch} a_k\tau_0 - 1], \quad a_k^{-1} \operatorname{sh} a_k\tau_0,$$

while when $\eta = 1$, $F_k = 0$.

With allowance for inequalities (2.2) and (2.9), we find the following estimate for the main solution (2.8)

$$\max |W(U, \tau, \eta)| \leqslant 2\varepsilon_0 \sum_1^{m_0} F_k(\eta), \ |U| \leqslant \varepsilon_0, \ 0 < \tau < \infty. \quad (2.10)$$

The function $\sum_{k=1}^{m_0} F_k(\eta)$ is also monotonically increasing when $\eta > 1$ and is equal to zero when $\eta = 1$. Considering this and inequality (2.10), as a lower bound for the critical force we can take the solution η_*^0 of the following equation

$$F(\eta_*^0) \equiv 2\sum_{k=1}^{m_0} F_k(\eta_*^0) = W_0/\varepsilon_0. \quad (2.11)$$

Equation (2.11) always has a solution at $\eta > 1$ by virtue of the above-mentioned properties of the left side of Eq. (2.11). This solution is actually the lower bound for the critical force.

In fact, for all $\eta < \eta_*^0 = N_*^0/N_e$, by virtue of the monotonic increase $F(\eta) < W_0/\varepsilon_0$. Then, with allowance for inequality (2.10)

$$\max |W(U, \tau, \eta)| \leqslant W_0, \ |U| \leqslant \varepsilon_0, \ 0 < \tau < \infty,$$

and by definition these values of η are subcritical, i.e., the critical force η_* does not fall within the interval $1 \leqslant \eta \leqslant \eta_*^0$. This means that $\eta_* > \eta_*^0$.

Equation (2.11) was used to construct the dependence of the lower bound for the critical force $\eta_*^0 = N_*^0/N_e$, shown in Fig. 1, on the dimensionless time of action of the pulse τ_0 for different values of the parameters a, $\sigma = W_0/\varepsilon_0$. Curves 1-6 correspond to a = 100, 100, 50, 50, 50, 50; σ = 400, 200, 400, 200, 100, 20.

It is evident from the figure that with a sufficiently long time of action of the pulse τ_0, $\eta_*^0 \to 1$. With short times, the value of η_*^0 is considerably greater than unity, i.e., the estimate obtained is not trivial, coincident with the critical force under static loading N_e. Rather, it is significantly greater than this force, which makes it possible to obtain a substantially higher permissible compressive force during shock loading than under static loading. Consequently, the structure can withstand larger loads than originally believed. A determination should be made of the boundaries of the parameters which, when approached, signify that the results obtained here have become unreliable.

At $\tau_* < 2$-3, the results may prove unreliable due to failure to account for the finite rate of propagation of the compressive force in the rod.

When $W_0/\varepsilon_0 < 10$, the results become unreliable due to representation (2.8) and the fact that the remaining part of the sum was ignored. On the other hand, with large values of W_0 the results become unreliable because we examined a linear equation of rod bending.

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METHODS OF SOLVING CONTACT THERMOELASTICITY PROBLEMS WITH ALLOWANCE FOR THE WEAR OF INTERACTING SURFACES

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1. Assume that a heavy cylindrical stamp is pressed into a rough, elastic (G, ν) layer with a large thickness, h. A force P, which is constant in time, is applied with the eccentricity e for each unit length of the stamp. The stamp moves at a constant velocity V along its generatrix; it is assumed that its area of contact with the layer has the width $2a(ha^{-1} \gg 1)$ and does not change in the course of time (see Fig. 1). This involves wear of the layer surface, which is accompanied by heat release in the region of contact. We assume that the stamp itself is not subject to wear. Coulomb friction forces arise in the region of contact [1, 2],

$$\tau_{yz} = (k_1 + k_2 T)q, \tag{1.1}$$

where k_1 and k_2 are constants, T is the temperature in the region of contact, and $q = q(x, t)$ is the contact pressure.

The condition of contact for solids 1 and 2 is written as follows:

$$v_1 + v_2 + v_3 = -[\delta(t) + \alpha(t)x - f(x)] \quad (|x| \leqslant a), \tag{1.2}$$

where v_1 is the displacement of the elastic layer's upper boundary due to the crushing of roughnesses, v_2 is the elastic deformation of the layer's surface, v_3 is the displacement of the y = 0 boundary of the layer due to its wear, $\delta(t) + \alpha(t)x$ is the rigid displacement of

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